GENERAL RADIO COMPANY

engineering department

INSTRUMENT NOTES

IN-102 1964

SERIES AND PARALLEL COMPONENTS OF IMPEDANCE

Users of General Radio Type 1608-A and 1650-A Impedance Bridges can choose to measure either series or parallel capacitance and either series or parallel inductance. This feature has apparently confounded some who regard the choice between C_s and C_p, or L_s and L_p, as an unnecessary complication in what should be a simple measurement.

The confusion — or rather, lack of information — about series and parallel parameters is nothing new. Writing in the January, 1946 *General Radio Experimenter*, W. Norris Tuttle noted that "Discussion of . . . series and parallel components, however, seldom appears in the elementary textbooks." Dr. Tuttle, a member of GR's engineering staff, went on to set the record straight on series and parallel components of impedance. Sixteen years later, for the benefit of a new generation who may not be clear on the series-parallel distinction, we offer the following reprint of Dr. Tuttle's excellent *Experimenter* article.

We have received lately a number of inquiries about the meanings of such terms as "series capacitance," "parallel capacitance," "series resistance," "parallel resistance," etc., as they are used in instruction books for General Radio bridges. Although most engineers think in terms of the series components of impedance, many types of problems, particularly those involving vacuum tubes, are more simply handled in terms of the parallel components. Certain bridge circuits give directly the series components of an impedance, while others can be arranged to give the parallel components, the choice depending on the intended application. Discussion of the relationship between the series and parallel components, however, seldom appears in the elementary textbooks.

That any impedance can be represented both ways is clear from the fact that measurements on it at a single frequency can determine only the relationship between the voltage across the impedance and the in-phase and quadrature components of the current flowing through it. Stated in terms of power engineering, a circuit element draws a certain amount of power at a particular value of power factor, and these two quantities completely define the effective impedance of the element for the conditions applying. It is sometimes convenient to represent the impedance as a pure resistance in series with a pure reactance, but it is very often more convenient to consider it as made up of a different value of resistance in parallel with a reactance. The two representations, however, are completely equivalent and either pair of components can be simply determined in terms of the other pair.

For example it will be seen that, in three cases shown in Figure 1, the first configuration of series elements would draw the same current, both in phase and magnitude as the second configuration, consisting of resistive and reactive elements in parallel. The two arrangements of each case are indistinguishable from each other by measurements made at their terminals at a fixed frequency.

The general relationship between the elements of the series and parallel arrangements can be simply found by equating the current drawn in the two cases.

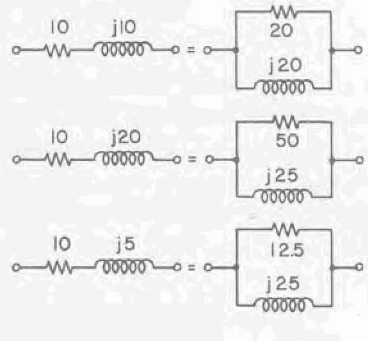


Figure 1. Examples of equivalent series and parallel circuits.

$$i = \frac{e}{R_s + jX_s} = \frac{e}{R_p} + \frac{e}{jX_p} \quad (1)$$

where R_s and X_s are the series components and R_p and X_p are the parallel components. Rationalizing and equating the real and imaginary terms,

$$\frac{R_s}{R_s^2 + X_s^2} = \frac{1}{R_p}$$

or (2)

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right)$$

$$\frac{X_s}{R_s^2 + X_s^2} = \frac{1}{X_p}$$

or (3)

$$X_p = X_s\left(1 + \frac{R_s^2}{X_s^2}\right)$$

The quantity X_s/R_s is the familiar Q or storage factor of an inductor or capacitor, and its reciprocal is the dissipation factor D, more frequency employed in describing the losses in capacitors. Substituting these quantities in Equations (2) and (3),

$$R_p = R_s\,(1 + Q^2) = R_s\left(1 + \frac{1}{D^2}\right) \quad (4)$$

$$X_p = X_s\left(1 + \frac{1}{Q^2}\right) = X_s\,(1 + D^2) \quad (5)$$

These equations give the parallel components of impedance directly in terms of the series components. The relationships, however, serve equally well when the series components are required and the parallel components are given, because the quantity Q or D can be determined directly from either the series or parallel components. Dividing (4) by (5),

$$\frac{R_p}{X_p} = \frac{R_s}{X_s}\,Q^2 = \frac{R_s}{X_s}\,\frac{1}{D^2}$$

or (6)

$$Q = \frac{1}{D} = \frac{X_s}{R_s} = \frac{R_p}{X_p}$$

so that Q can be determined immediately, whichever components are given, and used in Equations (4) and (5) to obtain the other components. A further simplification is that only one of the two Equations (4) and (5) need be employed with (6) to make the complete transformation. The three steps in each case are as follows:

Given R_s and X_s

(1) $Q = \dfrac{X_s}{R_s}$

(2) $R_p = R_s\,(1 + Q^2)$

(3) $X_p = \dfrac{R_p}{Q}$

Given R_p and X_p

(1) $Q = \dfrac{R_p}{X_p}$

(2) $R_s = \dfrac{R_p}{1 + Q^2}$

(3) $X_s = R_s Q$

If it is preferred to work in terms of dissipation factor the corresponding steps are:

Given R_s and X_s

(1) $D = \dfrac{R_s}{X_s}$

(2) $R_p = R_s\left(1 + \dfrac{1}{D^2}\right)$

(3) $X_p = R_p D$

Given R_p and X_p

(1) $D = \dfrac{X_p}{R_p}$

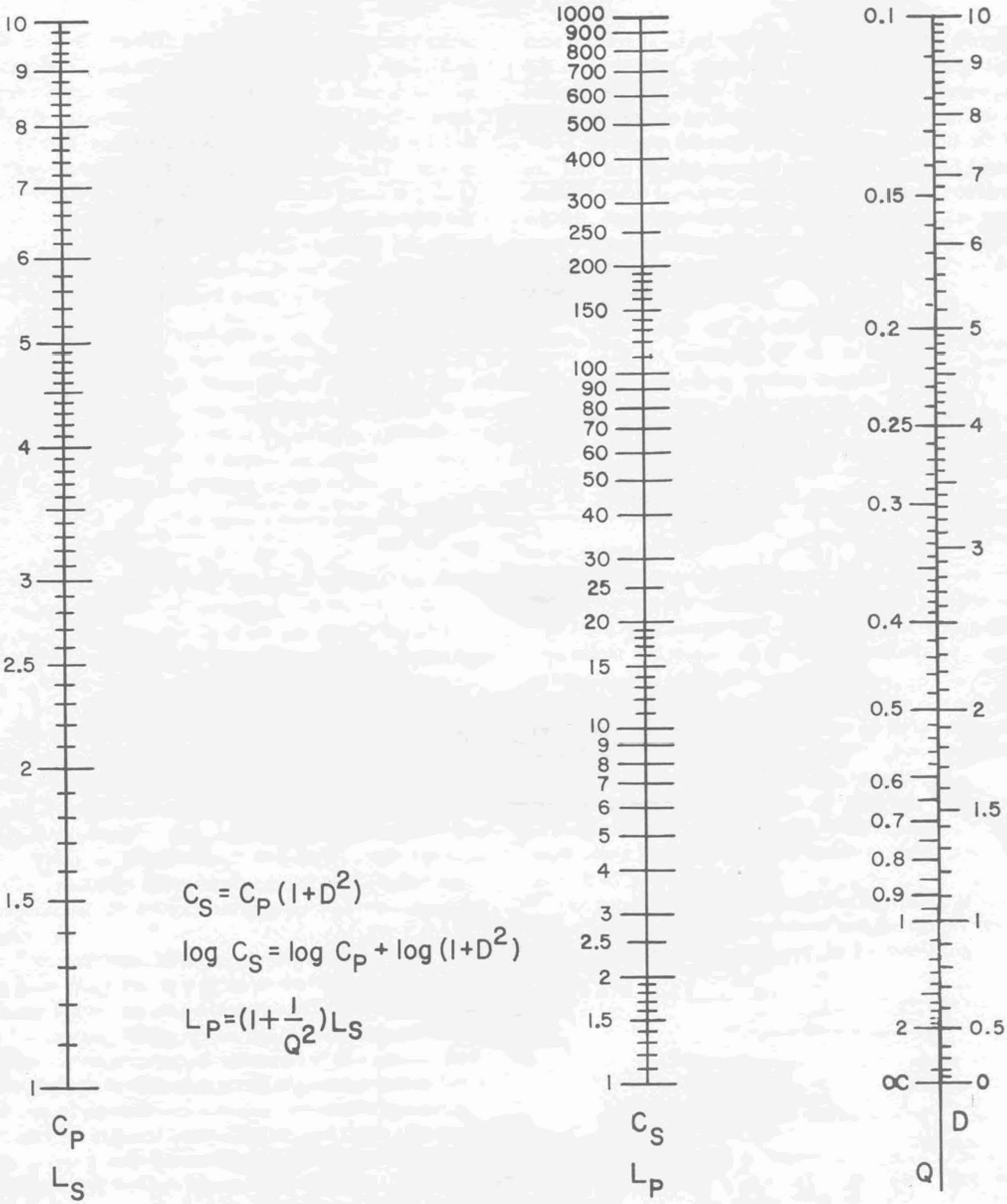
(2) $R_s = \dfrac{R_p}{1 + \dfrac{1}{D^2}}$

(3) $X_s = \dfrac{R_s}{D}$

It is seen that use of Q or D, which are associated with equal simplicity with either the series or parallel components, greatly facilitates the transformation. Since (6) is readily borne in mind, the only relation that need be remembered is that, as seen from (4), the ratio between the parallel and series resistances is the quantity $1 + Q^2$. It should be noted that the parallel resistance and parallel reactance are always greater than the corresponding series components. It is obvious that for large Q the series resistance must be small compared with the series reactance, but the parallel resistance must be large compared with the parallel reactance.

One of the simplest examples of the utility of the parallel impedance components is in parallel resonant

The nomograph below greatly simplifies the process of converting from series to parallel values (or vice versa) of inductance and capacitance, for values of dissipation factor up to 10 (Q down to 0.1). To illustrate use of the nomograph, assume a parallel capacitance of 2 μf, and a D of 7. A straight line connecting these two points is seen to cross the center (C_s) bar at 100. Therefore, the equivalent series capacitance is 100 μf.



$$C_S = C_P(1+D^2)$$

$$\log C_S = \log C_P + \log(1+D^2)$$

$$L_P = \left(1+\frac{1}{Q^2}\right)L_S$$

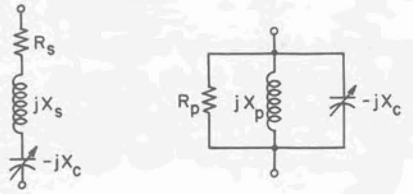


Figure 2. Series and parallel resonant circuits. The capacitance necessary to resonate with a given inductance will depend upon whether the elements are connected in series or in parallel.

circuits where the coil losses are high. It will be seen in Figure 2 that parallel resonance occurs when the capacitor reactance is exactly equal to the parallel reactance of the inductor, regardless of the coil losses. If the tuning capacitance for parallel resonance is determined from the series components of the coil impedance, on the other hand, the required value depends both on the resistance and on the reactance. In the series circuit the opposite applies and resonance occurs when the capacitor reactance is exactly equal to the series reactance of the inductor. Where the Q of the coil is high, the difference between its series and parallel reactance is negligible in ordinary applications. Even with a Q of 10 the difference is only one per cent. But for lower values of Q the difference rapidly increases. The parallel reactance of an inductor with a Q of 1 is twice the series reactance, so that only half the capacitance is required to tune it to resonance in a parallel circuit as in a series circuit.

— W. N. Tuttle

The foregoing article by Dr. Tuttle discusses series and parallel *reactance,* not series and parallel capacitance and inductance as such. The formulas C and L can be easily derived from those appearing in the article:

$$L_s = L_p \frac{Q^2}{1+Q^2} \quad = L_p \frac{1}{1+D^2}$$

$$L_p = L_s \frac{1+Q^2}{Q^2} \quad = L_s (1+D^2)$$

$$C_s = C_p (1+D^2)$$

$$C_p = C_s \left(\frac{1}{1+D^2}\right).$$

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OPERATING THE GENERAL RADIO TYPE 1133-A FREQUENCY CONVERTER WITH COUNTERS OF OTHER MAKES

The Type 1133-A Frequency Converter is designed for use with the GR Type 1130-A Digital Time and Frequency Meter and the Type 1153-A Digital Frequency Meter. It can, however, be operated with other 10-Mc counters or as a general-purpose frequency converter with other accessory equipment.

REFERENCE-FREQUENCY INPUT

The converter ordinarily requires a 5-Mc reference-frequency input (supplied by the counter) patched into a rear connector. Other reference-frequency sources can be used, however, as described below.

5 Mc/s

Any source of 5 Mc/s capable of supplying 15 mV or more into a 50-ohm load (e.g. 30 mV behind 50 ohm) can be used to drive the converter. Because of a narrow-band crystal filter in the converter, a lower-frequency source with a strong harmonic at 5 Mc/s can also be used.

100 kc/s, 200 kc/s, 500 kc/s

The Type 1153-P1 Frequency Multiplier, which plugs into the rear of the converter, multiplies a 100-kc reference-frequency input of 1-volt rms or greater (1-volt peak-to-peak for a square wave) to 5 Mc/s to operate the converter. The multiplier requires a supply voltage of +20 V at 8 mA. It will also operate with other input frequencies which are submultiples of 5 Mc/s, such as 200 kc/s and 500 kc/s.

1 Mc/s

If a 1-Mc signal does not have sufficient 5-Mc harmonic voltage to drive the converter, a fast-switching germanium or silicon diode can be connected in series with the reference-frequency input connector of the converter. Satisfactory diodes are the 1N994 and HHD5000 types. The diode can be conveniently mounted in a Type 874-X Insertion Unit, which can be plugged into the INPUT connector at the rear of the converter. This scheme works well with Beckman Instruments counters.

10 Mc/s (H-P SERIES 524 COUNTERS)

The reference-frequency circuits of the converter can be operated from a 10-Mc source of 100 mV or greater into 50 ohms if the first stage of the converter is rewired as outlined below.

1. Remove instrument from cabinet (see Instruction Manual page 13).
2. Remove shield from 10's Reference Frequency Generator section (remove 10 nuts, see page 14).
3. Clip out C401 and C404 (page 29).